# HOW LAST PLANNER MOTIVATES SUBCONTRACTORS TO IMPROVE PLAN RELIABILITY - A GAME THEORY MODEL 

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#### Abstract

There is an inherent relationship between subcontractors' labour resource allocation behaviour and the level of plan reliability they perceive. Under fixed-price or lump sum contracts, projects with low plan reliability can only be profitable for subcontractors when buffers of ready work are large enough to shield their productivity. A normal form game theory analysis can show that subcontractors will naturally tend to behave defensively whenever they perceive that plans are unreliable, resulting in unreliable labour allocation, and thus reducing plan reliability further, resulting in a vicious circle. The Last Planner System works to improve plan reliability. However, in order to achieve continuous improvement of the system, a rigorous model is needed to improve understanding of the mechanisms by which it affects labour resource allocation behaviour. The extended form game theory model presented in this research explains the relationship between project managers and subcontractors, and indicates at what levels of trust behaviour changes from competitive to collaborative. Ideas for enhancing construction procurement and production system design to make plans more reliable are discussed against the background of this theoretical explanation.


## KEY WORDS

Subcontracting, game theory, Last Planner, plan reliability.

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## INTRODUCTION

Achieving a high degree of plan reliability in construction is difficult because any one of the numerous prerequisite conditions for successful first-time completion of any work package may prove to be unreliable (Koskela 2000). However, the degrees of control that can be exercised over the preconditions, and the degree of certainty with which their reliability can be assessed, vary.

The Last Planner System (Ballard 2000) works to enhance reliability in three main ways: the 'look ahead planning' and 'make-ready' process, in which construction managers make work ready by ensuring that materials, information and equipment are available; filtering planned activities through the weekly work planning procedure to ensure that the preceding activities have been completed; and lastly, by seeking conscious and reliable commitment of labour resources by the leaders of the work teams involved.

The last facet is perhaps the most significant feature of the technique because it could be argued that the first two steps are reasonably obvious to experienced project managers and are commonly performed. It is this last step, which requires the conceptual leap from 'should' to 'can' (or 'push' to 'pull'), that requires schooling in lean thinking and lean construction. It focuses a spotlight on the inherent link between plan reliability and reliable allocation of resources, and recognizes that those actually performing the work are the most reliable predictors of labour capacity because they themselves allocate the labour resources.

Intuitively one can deduce that the more reliable a project work plan is over time, the more willing subcontractors will be to allocate resources more readily and suffice with smaller buffers, allowing WIP to be reduced and flow of work to be improved. However, a rigorous understanding of this relationship is needed.

Sacks (2004) introduced an economic model that explained why subcontractors in projects where labour resources are provided by subcontractors under fixed price or lump sum contracts require a high degree of certainty that work can be completed before committing resources. Because labour productivity is the key determinant of economic success or failure for the subcontractors, they prefer that large buffers of visibly ready work be accumulated ahead of their teams.

Sacks and Harel (2006) used the economic model as the basis for a game theory model of the relationship between project managers and subcontractors in demanding and allocating resources. The model showed that where plan reliability is low, fines or rewards are ineffective unless they are substantial in size and rigorously applied. The size of effective fines or rewards can be calculated for any probability distribution of confidence in plan reliability. A mathematical development of the conditions for changed behaviour under low plan reliability shows that the sub-optimal behavior is stable for a wide range of contract and pricing conditions.

On the other hand, a high degree of plan reliability, coupled with a high degree of confidence on the part of the subcontractor in it, leads to more predictable behavior in assigning resources. Analysis of the model provides a possible interpretation for the mechanism by which the Last Planner system improves plan reliability: by first increasing the confidence of each work team in the plan, which then has a second degree effect of improving the reliability in providing the right resources, which in turn improves plan reliability.

The following sections present Sacks and Harel's (2006) game theory model and then discuss its implications for the Last Planner system and for possible alternative and/or complementary measures that can be considered when designing construction procurement and production systems.

## GAME THEORY FORMULATION

The game theory formulation developed by Sacks and Harel (2006) models the allocation of resources at the start of each planning period in a project (typically each week). The players are the work planner (PM) and the subcontractor (SUB). Each makes 'moves', one after the other, through repeated cycles of the game. The work planner is a project management function in traditional construction systems (denoted 'PM') (later we will reconsider the identity of this role as it is redefined in the Last Planner system). In each round of the game, the PM sets the amount of work to be performed by each subcontractor (SUB) in each task $i$ in each period on the basis of the construction master plan. In response, each SUB evaluates the demand and the amount of work they perceive will actually become available, and then supplies the resources they deem appropriate.

The extensive form of game theory analysis (Osborne and Rubinstein 1994) is useful in this situation because the moves are sequential and because both PM and SUB have imperfect knowledge about the outcome in terms of the work that will actually be accomplished (i.e. they cannot predict with certainty how much work will be made available by the upstream contractors, or whether design changes, material delays, weather conditions or other factors will interrupt or slow work). The extensive form can be repeated in order to examine longterm strategies that develop as the parties respond to one another's previous actions and develop a relationship over time, which may facilitate cooperative behaviour (Lazar 2000).

The basic premises for the game theory analysis are that both the general contractor's project manager (PM) and the subcontractor's manager (SUB) behave rationally, which means that:

- the action chosen by the PM or the SUB is at least as good, according to his or her preferences, as every other available action (Fudenberg and Tirole 1991; Osborne 2004);
- they are both in a continuing conflict, as defined by Luce and Raiffa (1957);
- and they are both players in a non-cooperative game, which means that each is concerned only with his or her results (Gass 1985; Osborne and Rubinstein 1994).

Figure 3 details a typical round of the game model. The branches of the root node of the tree represent the range of possible results in terms of the amount of work that will actually be performed by the SUB. They are expressed using the ratio $q$ of work actually performed, $W_{A}$, to the work initially planned, $W_{P}$, where $q=W_{A} / W_{P}$. The probability of any particular value of $q$ occurring is described by a probability distribution, $\mathrm{P}[q]$, which is essentially a measure of plan reliability at the site. As such, it can be loosely compared to the PPC measure (Ballard 2000).

Extensive research covering a large sample size (Bortolazza et al. 2005) suggest that PPC values in the range of $80 \%$ to $90 \%$ are common after implementation of the Last Planner system. For the model analysis, four discrete values are used to represent the continuous function $\mathrm{P}[q]: 10 \%$ probability that $80 \%$ of the work planned for in any period will be possible, $20 \%$ that $90 \%$ will be possible, $50 \%$ chance of $100 \%$, and $20 \%$ likelihood that the work possible will exceed that planned by $10 \%$. The cumulative probability that at least $100 \%$ will be performed is $70 \%$, and the weighted average of the distribution is $98 \%$.


Figure 1. Extensive form game of subcontractor resource allocation.

The PM's possible moves are detailed at the second level from the left hand side of Figure 3 using the ratio $d$, which is the ratio of the work demanded, $W_{D}$, to the work he PM estimates will become available, $W_{P}$. The value $d$ is also modelled by discrete values: demand resources for less work than estimated ( $d=0.9$ ), exactly the amount estimated $(d=1)$ and more than estimated ( $d=1.1$ ). In response to the PM's request, the SUB can then elect to allocate fewer resources than required for the work demanded (setting $k=0.9$, where $k$ is the ratio of resources supplied to those demanded, exactly the amount required $(k=1)$ or more than demanded $(k=1.1)$. The latter strategy reflects a situation in which the SUB has resources available, and is willing to commit them in the hope that more work than expected will in fact become available and that they would be utilized profitably.

The utilities for each player are calculated at the end node of each branch of the tree. The utility for the PM is assumed to be the total amount of work actually completed in the planning period. When insufficient resources are allocated (i.e. when qe»dk), the work done is constrained by the quantity of resources available, and is proportional to $d k$; on the other hand, when $0 d » q<d k$, the work done is constrained by the availability of work and is therefore directly proportional to $q$. Thus the utility for the PM, $U_{P M}$, is given by $U_{P M}=\operatorname{Min}(q, d k)$.

The SUB's utility, $U_{S U B}$, is defined as the total income derived from the work done in the planning period, calculated on the basis of equation 1, which expresses a subcontractor's income, $c$, from work actually done on contract item $i, W_{A i}$, when resources are provided sufficient for performing an amount of work demanded, $W_{D i}$, under lump sum or unit price contracts, as developed in Sacks (2004) (the remaining terms are as follows: $U_{i}=$ the unit price for work on item $i, C_{M i}=$ material cost for one unit of work item $i, C_{S i}$ is the unit cost of labour and $\bar{r}$ is the standard work rate).
$I_{i}=W_{A i}\left(U_{i}-C_{M_{i}}\right)-W_{D_{i}} \frac{C_{S_{i}}}{\bar{r}}$
As the quantity of work done declines in relation to the quantity of work for which labour is allocated, the income will decline to zero and then become negative (Sacks 2004). In response to this potential outcome, a subcontractor may elect to provide fewer resources than those demanded by a project manager's plan of work, by the ratio $k$ defined above, and as expressed in equation 2 :
$I_{i}=W_{A i}\left(U_{i}-C_{M_{i}}\right)-k W_{D_{i}} \frac{C_{S_{i}}}{\bar{r}}$
In the general case, a subcontractor will attempt to maximize its income by trying to estimate what the value of $W_{A}$ is most likely to be, and then set $k=W_{A} / W_{D}$.

However, the work that can be done is constrained by the resources when $q \geq d k$. The SUB's utility (after dividing by $W_{D}$, which is constant, and substituting $q=W_{A} / W_{D}$ ) is therefore:
$U_{S U B}=\operatorname{Min}(q, d k)\left(U-C_{M}\right)-d k \frac{C_{S}}{\bar{r}}$
For example, as shown in Figure 1, if the work made available is $q=0.9$, the unit price is $U=100$, the material cost per unit of work is $C_{M}=30$, and the unit resource cost is $C_{S} / \bar{r}=60$ in any unit of currency, then the utilities for the case $d=1.0$ and $k=1.1$ are $U_{P M}=0.9$ and $U_{S U B}=-3.0$.

In extensive form games with probabilistic outcomes, the utilities are replaced by expected utilities to reflect the variability possible in the outcomes. In this case, since the utilities are all functions of $q$, the expected utility for each combination of PM and SUB strategies is the weighted average of the utilities for each possible result for $q$, weighted by its probability, which is given by the distribution $\mathrm{P}[q]$. Thus the PM's expected utility is:
$U_{P M}$ Expected $=\int_{0}^{d k} q P[q] d q+d k \int_{d k}^{\infty} P[q] d q=\int_{0}^{d k} q P[q] d q+p d k$
(where $p=\mathrm{P}[q>d k]$ ). Similarly, since the labour cost is independent of the amount of work performed, the SUB's expected utilities are computed as:
$U_{S U B}$ Expected $=\left(p d k+\int_{0}^{d k} q P[q] d q\right)\left(U_{i}-C_{M_{i}}\right)-k \frac{C_{S_{i}}}{\bar{r}}$
In practice, construction professionals would not estimate a continuous probability distribution, but rather use discrete values at significant intervals, expressed in the form described above and shown in Figure 1.

## RANGES OF ‘PM' AND ‘SUB' KNOWLEDGE AND BEHAVIOUR

Project planners' and subcontractors' behaviour is influenced by the degree of knowledge they have about the amount of work that will actually become available, which cannot be known with certainty at the start of each planning period. As such, their imperfect knowledge must be accounted for in solving the game theory model. A two-dimensional range of possible knowledge is postulated, with each axis representing the degree of knowledge of the PM and the SUB respectively, as shown in Figure 2. The PM's axis is the degree of plan reliability of the project: when plan reliability is low, the PM has uncertain information, and conversely, when plan reliability is high, the PM's knowledge is more certain. The PM is assumed to function within this range, between the extrema of no prior knowledge (Case A in Figure 2) and perfect prior knowledge (Case B in Figure 2).


Figure 2. Ranges of PM and SUB function for subcontractor resource allocation.
On the other hand, the SUB's knowledge cannot be simply represented by the plan reliability, because, in traditional systems, the SUB is called upon to provide the resources as demanded by the PM, and the SUB has imperfect knowledge about the PM's strategy (i.e. has the PM demanded more, exact or less resources than they have estimated will actually be needed?). The SUB must consider to what degree the PM can be trusted and may attempt to gather independent information about the likely work availability in each coming period. Thus the SUB functions along an axis between two ideal extrema: no prior knowledge (Case B in Figure 2) and full prior knowledge (Case C in Figure 2).

The extensive form game can be adapted to solve for each of the cases A, B and C, using 'information sets'. These situations of imperfect information are denoted in Figure 3 by the dashed lines to the left of the PM's and the SUB's nodes. The three cases are described in Table 2, which also lists the equilibrium ${ }^{3}$ situations that result when they are solved. Readers concerned with the details of the solution procedures are referred to Sacks and Harel (2006); this paper focuses on their interpretation in terms of the Last Planner system. Table 1 provides an example of the solution matrix for one of the three cases solved (case A).

Unfortunately, neither PMs nor SUBS have full control over all of the conditions needed to make resource requests consistently reliable, which means that cases B and C are ideal and cannot be achieved completely in practice. Similarly, no projects function in complete chaos, where the plan is entirely unpredictable, as represented by Case A. Thus real projects are assumed to function in the zone outlined in Figure 2.

Analysis of the game theory calculation results shows that the cooperative equilibrium, where exact resources are demanded and exact resources are provided, only occurs when the information situation of the participants approaches Case C. On the other hand, when neither participant has reliable information, both must resort to defensive, competitive strategies to reach equilibrium at their 'least bad' result. Because defensive behaviour on the part of the SUB includes provision of fewer resources than demanded, the ability of a PM to achieve plan reliability is harmed, creating a stable and self-perpetuating lose-lose situation.

The existence of such conditions in actual projects was explored in a series of structured interviews with 57 project managers and subcontractor works supervisors functioning in conventionally managed housing projects (Harel and Sacks 2006). In response to the question "When you demand resources from a subcontractor, to what degree do you exaggerate your report of the amount of work that will actually be available?" $48.3 \%$ of the respondents confirmed that they exaggerate their demands by at least $20 \%$. On the other hand, in response to the question "What proportion of the work promised by the project manager do you believe will actually be made ready?", more than $85 \%$ of the subcontractors interviewed believed that less than $80 \%$ of the work would be made ready. The average amount of work ready expected was only $60 \%$, with a standard deviation of $19.7 \%$.

[^1]Table 1. Normal form solution for Case A. The equilibrium is indicated.

| Subcontractor <br> strategies | Provide fewer <br> $\mathrm{k}=0.9$ |  | Provide exact <br> $\mathrm{k}=1.0$ |  | Provide more <br> $\mathrm{k}=1.1$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Manager <br> strategies | PM | SUB | PM | SUB | PM | SUB |
| Demand less <br> $\mathbf{d = 0 . 9}$ | 0.809 | 8.0 | 0.890 | 8.3 | 0.953 | 7.3 |
| Demand exact <br> $\mathbf{d}=1.0$ | 0.890 | 8.3 | 0.960 | 7.2 | 0.980 | 2.6 |
| Demand more <br> $\mathbf{d}=1.1$ | 0.953 | 7.3 | 0.980 | 2.6 | 0.980 | -4.0 |



Figure 3. Extensive form game of subcontractor resource allocation.

Table 2: Extensive form game cases and results.

| Case | Description | Solution Space | Equilibrium Solutions |
| :---: | :---: | :---: | :---: |
| A | Neither the PM nor the SUB has any knowledge of the probability distribution of $q$, i.e. neither can predict how much work will be possible. | This case reduces to the normal form with a $3 \times 3$ matrix of possible solutions, for each of the three strategies for the PM (demand less, exact or more resources than needed) and the three strategies of the SUB (provide less, exact or more resources than demanded). | This form has a perfect equilibrium, which is the strategy pair: PM demand more; SUB provide fewer. The equilibrium is insensitive to variation of the profit margin, and relatively insensitive to fines that may be applied. The solution matrix is shown in ¡Error!Argumento de modificador desconocido.. |
| B | The PM has perfect knowledge of $q$, but the SUB has none; the information set in front of the PM is removed, but that in front of the SUB remains. | This case has $27 \times 3=81$ possible solutions, because the PM has nine strategies for each of the three situations that may arise in terms of the work that becomes available and the SUB's response. The SUB has just three strategies, as before. | This case has two significant equilibria. The first occurs for the PM demanding more work than estimated in every situation and the SUB providing fewer resources. A second equilibrium, which is part of a mixed strategy, occurs when the PM demands exact work when $\mathrm{q}=0.9$ or $\mathrm{q}=1.0$ and more when $\mathrm{q}=1.1$, and the SUB provides exact resources. |
| C | Both have full knowledge of the work to be made possible, which also implies that the SUB is fully aware of the PM's possible strategies. In this form both information sets are removed. | Here there are 27 strategies available for each of the PM and the SUB, because their actions are dependent on one another's actions and on the actual work availability outcomes. The solution matrix has $27 \times 27$ cells. | Here too there are two main equilibria. The first, similar to those in the previous cases, occurs when the PM demands more work in every case, and the SUB provides fewer resources in very case. The second occurs when exact resources are both demanded and allocated in every case. The numerical differences between them are very small. |

## IMPACT OF THE LAST PLANNER SYSTEM

The Last Planner has three impacts that are relevant in terms of the game theory model:

1) The degree of plan reliability is made transparent to all participants in the process, by means of the PPC (percent plan complete) measure.
2) Plan reliability is assumed to be improved.
3) Subcontractors are given direct control over the work assignment process itself through the weekly work planning meeting in as far as their own supervisors are considered 'last planners', which means that their knowledge of the work allocation strategy is likely to be at least as good as that of the project management function itself.

Improved plan reliability, and no less improved knowledge of it, moves project managers along the horizontal axis from Case $A$ to Case $B$ (with reference to Figure 2).

These two impacts, together with improved knowledge of the work allocation strategy, move subcontractors not only along the axis from Case A toward Case B, but also along the vertical axis in the direction from Case B toward Case C. Thus the game theory model suggests that use of the Last Planner system should encourage movement to an equilibrium of more cooperative behaviour than possible without it.

Cooperative behaviour in this context means increased tendency to demand exact resources required and to provide exactly the resources demanded. In turn, cooperative behaviour of this nature improves plan reliability for subsequent trades, raising the overall degree of plan reliability for the project. The mechanism of the system may thus be considered to be to facilitate movement toward stable modes of cooperative behaviour.

The Last Planner functions at the level of production control. Other approaches to engendering cooperative behaviour, such as partnering (Fisher and Green 2001; Rahman and Kumaraswamy 2004) function at the level of organizational and personal relationships. Their ability to achieve sustainable improvement is dependent on maintaining cooperative behaviour. In terms of the game theory model, as long as the basic utility functions remain unchanged, behaviour within systems where partnering arrangements are initiated is likely to regress toward the original 'lose-lose' equilibrium solutions inherent in the system. Examples of this effect have been reported (Lazar 2000; Packham et al. 2001), in which the basic interests of subcontractors remain unchanged, and the effects of partnering are felt in the short-term only. A possible implication is that if Last Planner is not implemented consistently and completely (i.e. both pull flow control using Last Planner meetings and consistent and transparent reporting of PPC), then its impact is also likely to be transient.

## CONCLUSIONS

The game theory model supports the conclusions reached in earlier research reported to the IGLC (Sacks 2004), in which an economic model of the motivation of subcontractors working under lump sum or unit price contracts in construction was developed. It explained the tendency of subcontractors to provide fewer resources than demanded when there is uncertainty concerning the amount of work that will be performed in any given planning period. The game theory formulation underlines the importance of plan reliability as a key factor influencing the subcontractors' behaviour, and shows that cooperative behaviour can only be expected where plans are reliable and subcontractors believe that they have full knowledge of a project managers' strategy.

Under the Last Planner System, the degree of plan reliability is made transparent to all (through reporting of the PPC), and presumed to be higher than in projects run using push control methods. Furthermore, subcontractors are not only made privy to the work allocation strategy of the project management function, but are indeed given control over it through the weekly work planning meetings. Thus in effect, application of the system moves construction projects toward the idealized conditions of Case C , as defined above. This makes the likelihood of a stable equilibrium of cooperative behaviour much higher than for traditionally managed projects.

The theoretical model is intended to enable consideration of additional steps that can be taken to improve plan reliability by manipulating the basic motivations of subcontractors in order to improve the reliability of their resource allocations. Possible ideas may include changes to the contractual terms under which subcontractors are employed, and which govern the utility functions (such as reserving part of the total remuneration for covering capacity rather
than product), which could obviate or soften the harmful effects of independent local optimization and provide managers extended ability to achieve overall system efficiencies).

Miller et al. (2002) argue that harmonization between contractors and subcontractors is a prerequisite for lean construction. While mutual cooperation and partnering arrangements can undoubtedly enhance construction performance, the model suggests that it should be possible to engender behaviour that enhances workflow stability at the project level through effective production management.

At this stage, the model uses only the economic aspect of the utility function for each participant. As explained by Harel and Sacks (2006), additional aspects and motivations, such as cash flow considerations, the promise of future work, reputation and others, should be incorporated in order to extend the applicability of the theoretical model.

The game theory model, and its interpretation in terms of the impact of the Last Planner system, suggests that there should be strong correlation between PPC and the behaviour of subcontractors in allocating resources. Testing for this relationship in empirical research, covering projects involving subcontractors operating under lump sum or unit price contracts and managed using both traditional and Last Planner procedures, is needed to validate or negate the model.

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[^1]:    3 In game theory, a Nash equilibrium can be defined as «an optimal collective strategy in a game involving two or more players, where no player has anything to gain by changing only his or her own strategy. If each player has chosen a strategy and no player can benefit by changing his or her strategy while the other players keep theirs unchanged, then the current set of strategy choices and the corresponding payoffs constitute a Nash equilibrium.» Wikipedia. (2005). Nash Equilibrium. [http://en.wikipedia.org/wiki/Nash_equilibrium](http://en.wikipedia.org/wiki/Nash_equilibrium), last accessed 4 December 2005.

